Last Name _____ First Name _____ Identification Number _____

Degree in Economics, Business and Financial Markets
English Laboratory of Mathematics for Economics – Prof. E. Colantonio
Test of xx/xx/xxxx

Time available: 90 minutes

You must answer at least 5 (of 9) questions correctly to pass the Test (wrong answers do not penalize)

Given the function $y = \sqrt{x+4}$

1. State the domain

a. $x \geq -4$ **b.** $x \geq 4$ **c.** $x \leq -4$ **d.** $x \leq 4$

2. Is the function even, odd, both or neither?

a. even **b.** odd **c.** both **d.** neither

3. Find the points at which the function cuts the x and y axes

a. $(4;0);(0;2)$ **b.** $(-4;0);(0;2)$ **c.** $(-4;0);(0;-2)$ **d.** $(4;0);(0;-2)$

4. y is positive if…

a. $x > -4$ **b.** $x > 4$ **c.** $x < -4$ **d.** $x < 4$

5. Find any asymptotes

a. $x = 0$ **b.** $y = -4$ **c.** $y = x - 4$ **d.** none

6. Find any minima (min) and maxima (MAX) for the function

a. MAX at $(4;0)$ **b.** min at $(-4;0)$ **c.** min at $(0;-2)$ **d.** none

7. Tom spends all his 100 weekly income on two goods, X and Y. His utility function is given by $U(X,Y) = XY$. If $P_X = 4$ and $P_Y = 10$, how much of each good should he buy?

a. $X = 12.5; Y = 5$ **b.** $X = 12; Y = 5.2$ **c.** $X = 11; Y = 5.6$ **d.** $X = 14; Y = 4.4$

8. Consider the following two-player game with normal form and find all Nash equilibria

		Player 2	
		L	**R**
Player 1	**T**	**6** **7**	**5** **0**
	B	**0** **2**	**3** **4**

a. (T, L); (B, R) **b.** (B, R) **c.** (T, L) **d.** (T, R)

9. Consider the following financial operation and find the Net Present Value when the cost of capital is 10%

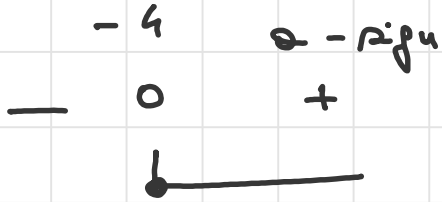
Year	Cash Flows
0	**-2000**
1	**1000**
2	**3000**
3	**2000**

a. 3040.12 **b.** 2891.06 **c.** 2509.68 **d.** 4000.00

$y = \sqrt{x+4}$

- Domain: radicand ≥ 0 $\Rightarrow$

$x + 4 = 0 \Rightarrow x = -4$



Domain: $x \geq -4$

- Even or Odd f. ? Asymmetric Domain $\Rightarrow$

NEITHER

- Intersections

if $x = 0 \Rightarrow y = \sqrt{0+4} \Rightarrow y = 2 \quad (0;2)$

if $y = 0 \Rightarrow$ rad. $= 0 \Rightarrow$ root already found

$(0;2)$; $(-4;0)$

- y positive if ... come il domain, root esclusa

$y > 0$ if $x > -4$

- Asymptotes

$\lim_{x \to +\infty} \sqrt{x+4} = \infty \Rightarrow$ oblique asymptote?

$y = ax + b \Rightarrow a = \lim_{x \to +\infty} \frac{f(x)}{x} \Rightarrow$

$\lim_{x \to +\infty} \frac{\sqrt{x+4}}{x} \Rightarrow \sqrt{\frac{x+4}{x^2}} = \frac{\infty}{\infty} \Rightarrow$

$\Rightarrow \lim_{x \to +\infty} \sqrt{\frac{x^2\left(\frac{1}{x} - \frac{4}{x^2}\right)}{x^2 (1)}} = \sqrt{\frac{0}{1}} = 0$

a deve essere finito e $\neq 0 \Rightarrow$ no oblique asymptote

stessa cosa per $x \to -\infty$ ...

HOME

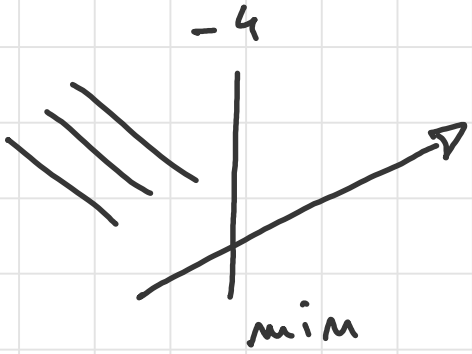
- MAX and min

$D\left(\sqrt{x+4}\right) \Rightarrow D(x+4)^{1/2} \Rightarrow$

$\Rightarrow \frac{1}{2}(x+4)^{1/2-1} \cdot 1 \Rightarrow \frac{1}{2}\frac{1}{\sqrt{x+4}}$

$+ \quad \hookrightarrow +$ nel dominio

Funzione sempre crescente



7. Tom spends all his 100 weekly income on two goods, X and Y. His utility function is given by $U(X,Y) = XY$. If $P_X = 4$ and $P_Y = 10$, how much of each good should he buy?

$$L = xy - \lambda(4x + 10y - 100) \Rightarrow$$

$$\Rightarrow \quad \frac{\partial L}{\partial x} = 0 \Rightarrow \quad y - 4\lambda = 0$$

$$\frac{\partial L}{\partial y} = 0 \Rightarrow \quad x - 10\lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow \quad 4x + 10y - 100 = 0 \quad \text{(V. di B.)}$$

Dalle prime due equazioni otteniamo

$$\frac{y}{x} = \frac{4\lambda}{10\lambda}$$

Dividendo membro a membro

$$\frac{y}{x} = \frac{4}{10} \Rightarrow \quad y = \frac{2}{5}x \quad \Rightarrow \text{cont. nel V. di B.}$$

$4X + \frac{10^2}{5} \cdot \frac{2}{5} X = 100 \Rightarrow 8X = 100 \Rightarrow$

$\Rightarrow X = 12,5$

$Y = \frac{2}{5} X \Rightarrow Y = \frac{2}{5} \cdot 12,5 \Rightarrow Y = 5$

8. Consider the following two-player game with normal form and find all Nash equilibria

		Player 2	
		L	**R**
Player 1	**T**	(6) **7**	**0** **5**
	B	**2** **0**	(4) (3)

Best reply ...

2 Nash equilibria $\Rightarrow$ (T;L) ; (B;R)

9. Consider the following financial operation and find the Net Present Value when the cost of capital is 10%

Year	Cash Flows
0	-2000
1	1000
2	3000
3	2000

$NPV = -2000 + \frac{1000}{1+0,1} + \frac{3000}{(1+0,1)^2} + \frac{2000}{(1+0,1)^3} =$

$= -2000 + 909,09 + 2479,34 + 1502,63$

$= 2891,06$