Last Name _____ First Name _____ Identification Number _____

Degree in Economics, Business and Financial Markets
English Laboratory of Mathematics for Economics – Prof. E. Colantonio
Test of xx/xx/xxxx

Time available: 90 minutes

You must answer at least 5 (of 9) questions correctly to pass the Test (wrong answers do not penalize)

Given the function $y = \sqrt{x^2 - x - 2}$

1. State the domain

a. $x \leq 1 \cup x \geq 2$ **b.** $x \leq -1 \cup x \geq 4$ **c.** $x \leq -1 \cup x \geq 2$ **d.** $x < 1 \cup x > 2$

2. Is the function even, odd, both or neither?

a. even **b.** odd **c.** both **d.** neither

3. Find the points at which the function cuts the x and y axes

a. $(-1; 0); (2; 0)$ **b.** $(-2; 0); (1; 0)$ **c.** $(-2; 0); (2; 0)$ **d.** $(-1; 0); (4; 0)$

4. y is positive if…

a. $x < 1 \cup x > 2$ **b.** $x < 1 \cup x > 4$ **c.** $x < -1 \cup x > 0$ **d.** $x < -1 \cup x > 2$

5. Find any asymptotes

a. $y = x + 1/2$, $y = -x + 1/2$ **b.** $y = x - 1/2$, $y = -x + 1/2$ **c.** $y = x - 1/2$, $y = -x - 1/2$ **d.** $y = x - 2$, $y = -x + 2$

6. Find any minima (min) and maxima (MAX) for the function

a. $min\ at\ (1)$, $min\ at\ (2)$ **b.** $min\ at\ (-1)$, $min\ at\ (2)$ **c.** $min\ at\ (-1)$, $MAX\ at\ (2)$ **d.** $MAX\ at\ (-1)$, $min\ at\ (2)$

7. Tom spends all his 96 weekly income on two goods, X and Y. His utility function is given by $U(X, Y) = XY^3$. If $P_X = 6$ and $P_Y = 12$, how much of each good should he buy?

a. $X = 2; Y = 7$ **b.** $X = 3; Y = 6.5$ **c.** $X = 4; Y = 6$ **d.** $X = 5; Y = 5.5$

8. Consider the following two-player game with normal form and find all Nash equilibria

		Player 2	
		C	**D**
Player 1	**A**	**15** **15**	**0** **-50**
	B	**-50** **0**	**10** **10**

a. (A, C); (B, D) **b.** (A, C) **c.** (B, D) **d.** (A, C); (B; C)

9. Consider the following financial operation and find the Net Present Value when the cost of capital is 10%

Year	Cash Flows
0	**-2000**
1	**-1000**
3	**3000**
5	**2000**

a. 586.70 **b.** 682.20 **c.** 432.85 **d.** 2000.00

$$y = \sqrt{x^2 - x - 2}$$

- Domain => radicand ≥ 0

$\Delta = 1 + 8 = 9 > 0$ => 2 real solutions

$x_{1/2}$: $\dfrac{1 \pm \sqrt{9}}{2}$ → $x_1 = \dfrac{1-3}{2} = -1$

→ $x_2 = \dfrac{1+3}{2} = 2$

a-sign		-1		2		a-sign
	+	0	−	0	+	

Domain: $x \leq -1 \cup x \geq 2$

- Even or odd f.? Asymmetric domain => NEITHER
- Intersections

if $x = 0$ => not in the Domain

if $y = 0$ => rad $= 0$ => roots already found

$(-1; 0)$; $(2; 0)$

- y positive if ...

come il dominio, senza roots!

$y > 0$ if $x < -1 \cup x > 2$

- Asymptotes

$$\lim_{x \to +\infty} \sqrt{x^2 - x - 2} = \sqrt{x^2 \left(1 - \frac{1}{x} - \frac{2}{x^2}\right)} = +\infty$$

($\frac{1}{x} \to 0$, $\frac{2}{x^2} \to 0$)

Oblique asymptote? $y = ax + b$

$$a = \lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to +\infty} \frac{\sqrt{x^2 - x - 2}}{x} \Rightarrow \frac{\infty}{\infty}$$

$$= \dots = \frac{\sqrt{x^2}}{x}$$

Remind $\sqrt{x^2} = |x|$

$|x|$ and $\begin{cases} x \text{ se } x > 0 \\ -x \text{ se } x < 0 \end{cases}$

$$\Rightarrow \lim_{x \to +\infty} \frac{|x|}{x} = \frac{x}{x} = 1 \to a$$

$$b = \lim_{x \to +\infty} f(x) - ax = \sqrt{x^2 - x - 2} - 1x = \infty - \infty$$

Coniugato

$$\lim_{x\to+\infty}\left(\sqrt{\quad}-x\right)\left(\frac{\sqrt{\quad}+x}{\sqrt{\quad}+x}\right)=$$

$$=\frac{\cancel{x^2}-x-2\ -\cancel{x^2}}{\sqrt{x^2-x-2}+x}=\dots=\frac{-x-2}{\sqrt{x^2}+x}=$$

$$=\frac{-x-2}{|x|+x}=\frac{-x-2}{x+x}=\frac{-x-2}{2x}=\frac{\infty}{\infty}$$

$$\Rightarrow\lim_{x\to+\infty}\frac{\cancel{x}\left(-1-\frac{2}{x}\right)}{\cancel{x}\,(2)}=-\frac{1}{2}\to b$$

$y=x-\frac{1}{2}$ obliqua asymptote

$$\lim_{x\to-\infty}\sqrt{x^2-x-2}=+\infty$$

obliqua asymptote?

$$a=\lim_{x\to-\infty}\frac{\sqrt{x^2-x-2}}{x}=\frac{\sqrt{x^2\left(1-\underset{\to0}{\cancel{\frac{1}{x}}}-\underset{\to0}{\cancel{\frac{2}{x^2}}}\right)}}{x}=$$

$$=\frac{\sqrt{x^2}}{x}=\frac{|x|}{x}=\frac{-x}{x}=-1\to a$$

$$b = \lim_{x \to -\infty} \sqrt{x^2 - x - 2} - (-1)x = \sqrt{x^2 - x - 2} + x$$

$= \infty - \infty \Rightarrow$ Coniugato

$$\lim_{x \to -\infty} \left(\sqrt{\quad} + x \right)\left(\frac{\sqrt{\quad} - x}{\sqrt{\quad} - x} \right) =$$

$$= \frac{\cancel{x^2} - x - 2 \ \cancel{- x^2}}{\sqrt{x^2 - x - 2} - x} = \dots = \frac{-x - 2}{\sqrt{x^2} - x} =$$

$$= \frac{-x - 2}{|x| + x} = \frac{-x - 2}{-x - x} = \frac{-x - 2}{-2x} = \frac{\infty}{\infty}$$

$$\Rightarrow \lim_{x \to -\infty} \frac{x\left(-1 - \frac{2}{x}\right)}{x(-2)} \quad (\tfrac{2}{x} \to 0) = \frac{1}{2} \to b$$

$y = -x + \frac{1}{2}$ obliqua asymptote

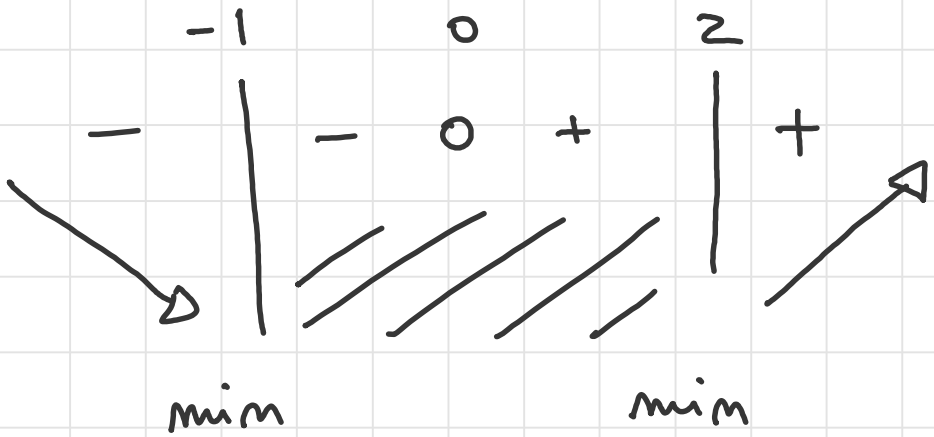
- MAX and min

$$D\left(\sqrt{x^2 - x - 2}\right) = D\left(x^2 - x - 2\right)^{1/2} =$$

$$= \frac{1}{2}\left(x^2 - x - 2\right)^{1 - 1/2} \cdot 2x = \frac{1}{\cancel{2}} \frac{\cancel{2}x}{\sqrt{x^2 - x - 2}}$$

↳ + in the Domain

$X = 0$



7. Tom spends all his 96 weekly income on two goods, X and Y. His utility function is given by $U(X,Y) = XY^3$. If $P_X = 6$ and $P_Y = 12$, how much of each good should he buy?

a. $X = 2; Y = 7$ **b.** $X = 3; Y = 6.5$ **c.** $X = 4; Y = 6$ **d.** $X = 5; Y = 5.5$

$$L = xy^3 - \lambda(6x + 12y - 96)$$

$$\begin{cases} \frac{\partial L}{\partial x} = 0 \Rightarrow & y^3 - 6\lambda = 0 \\ \frac{\partial L}{\partial y} = 0 \Rightarrow & 3xy^2 - 12\lambda = 0 \\ \frac{\partial L}{\partial \lambda} = 0 \Rightarrow & 6x + 12y = 96 \quad (\text{V. di B.}) \end{cases}$$

Dalle prime due equazioni ricavo

$$\frac{y^3}{3xy^2} = \frac{6\lambda}{12\lambda}$$

Divido membro a membro

$$\frac{y}{3x} = \frac{1}{2} \Rightarrow y = \frac{3}{2}x \rightarrow \text{sostituisco nel V. di B.}$$

$$6x + \cancel{12}^{6} \cdot \frac{3}{\cancel{2}} x = 96 \Rightarrow 24x = 96 \Rightarrow x = 4$$

$$y = \frac{3}{\cancel{2}} \cdot \cancel{4}^{2} \Rightarrow y = 6$$

8. Consider the following two-player game with normal form and find all Nash equilibria

		Player 2	
		C	D
Player 1	A	(15) 15	0 / -50
	B	-50 / 0	(10) 10

Best Reply ...

There are 2 Nash equilibria : (A; C) ; (B; D)

PLAYER 1

if P_2 plays C => P_1 prefers A (15) to B (0)
" " " D => " " B (10) to A (-50)

PLAYER 2

if P_1 plays A => P_2 prefers C (15) to D (0)
" " " B => " " D (10) to C (-50)

there are 2 Nash equilibria : (A, C) and (B, D), where the payoffs are both highlighted

9. Consider the following financial operation and find the Net Present Value when the cost of capital is 10%

Year	Cash Flows
0	-2000
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3	3000
5	2000

$$NPV = -2000 - \frac{1000}{(1+0,1)} + \frac{3000}{(1+0,1)^3} + \frac{2000}{(1+0,1)^5} =$$

$$= -2000 - 909,09 + 2253,94 + 1241,84 =$$

$$= 586,69$$