Last Name _____ First Name _____ Identification Number _____

Degree in Economics, Business and Financial Markets
English Laboratory of Mathematics for Economics – Prof. E. Colantonio
Test of xx/xx/xxxx

Time available: 90 minutes

You must answer at least 5 (of 9) questions correctly to pass the Test (wrong answers do not penalize)

Given the function $y = \frac{x^2+2x-3}{x-2}$

1. State the domain

a. $x \geq 2$ **b.** $x \in R - \{-2\}$ **c.** $x \in R$ **d.** $x \in R - \{2\}$

2. Is the function even, odd, both or neither?

a. even **b.** odd **c.** both **d.** neither

3. Find the points at which the function cuts the x and y axes

a. $(0; -3/2); (-3; 0); (1; 0)$ **b.** $(0; 3/2); (3; 0); (1; 0)$ **c.** $(0; 3/2); (-3; 0); (1; 0)$ **d.** $(0; 3/2); (-3; 0); (2; 0)$

4. y is positive if…

a. $-3 < x < 0 \cup x > 2$ **b.** $-2 < x < 1 \cup x > 2$ **c.** $-3 < x < 1 \cup x > 2$ **d.** $-3 < x < 1 \cup x > 3$

5. Find any asymptotes

a. $x = -2$, $y = x - 4$ **b.** $x = 2$, $y = x + 8$ **c.** $x = 2$, $y = x + 4$ **d.** $x = 2$, $y = x - 4$

6. Find any minima (min) and maxima (MAX) for the function

a. $MAX\ at\ (2+\sqrt{5})$, $min\ at\ (2-\sqrt{5})$ **b.** $MAX\ at\ (2-\sqrt{5})$, $min\ at\ (2+\sqrt{5})$ **c.** $MAX\ at\ (2-\sqrt{20})$, $min\ at\ (2+\sqrt{20})$ **d.** $MAX\ at\ (2+\sqrt{20})$, $min\ at\ (2-\sqrt{20})$

7. Tom spends all his 72 weekly income on two goods, *X* and *Y*. His utility function is given by $U(X,Y) = 6X^{1/3}Y^{2/3}$. If $P_X = 8$ and $P_Y = 2$, how much of each good should he buy?

a. $X = 2; Y = 28$ **b.** $X = 3; Y = 24$ **c.** $X = 4; Y = 20$ **d.** $X = 5; Y = 16$

8. Consider the following two-player game with normal form and find all Nash equilibria

		Player 2		
		C	**D**	**E**
Player 1	**A**	**3** **7**	**8** **5**	**5** **3**
	B	**6** **8**	**4** **6**	**3** **1**

a. (A, C); (B, D) **b.** (A, C) **c.** (B, C) **d.** (A, C); (B; C)

9. Consider the following financial operation and find the Net Present Value when the cost of capital is 5%

Year	Cash Flows
0	**-3000**
1	**-1000**
2	**5000**
4	**2000**

a. 1672.23 **b.** 1589.17 **c.** 2228.17 **d.** 3000.00

$$y = \frac{x^2 + 2x - 3}{x - 2}$$

- Domain => $\frac{N}{D \rightarrow \neq 0}$

$x - 2 = 0 \Rightarrow x = 2$

Domain: $x \in \mathbb{R} - \{2\}$

- Even or Odd f.? Asymmetric domain ...

NEITHER

- Intersections

if $x = 0 \Rightarrow y = \frac{0 + 0 - 3}{0 - 2} = \frac{3}{2}$ $(0; 3/2)$

if $y = 0 \Rightarrow \frac{N \rightarrow}{D} = 0 \Rightarrow x^2 + 2x - 3 = 0$

$\Delta = 4 + 12 = 16 > 0$

=> 2 real solutions: $x_{1/2}: \frac{-2 \pm \sqrt{16}}{2}$

$x_1 = \frac{-2-4}{2} = -3$

$x_2 = \frac{-2+4}{2} = 1$

$(0; 3/2)$; $(-3; 0)$; $(1; 0)$

- y positive if ... roots already found

		-3		1		2	
N	+	0	−	0	+	+	+
D	−	−	−	−	−	0	+
$f(x)$	−	0	+	0	−	∄	+

$y > 0$ if $-3 < x < 1 \quad \cup \quad x > 2$

- Asymptotes

$$\lim_{x \to 2} \frac{x^2+2x-3}{x-2} = \frac{4+4-3}{2-2} = \frac{5}{0} \Rightarrow \frac{\text{non zero}}{0}$$

$\Rightarrow$ $x = 2$ vertical asymptote

$$\lim_{x \to +\infty} \frac{x^2+2x-3}{x-2} = \frac{\infty}{\infty} \Rightarrow$$

$$\Rightarrow \lim_{x \to +\infty} \frac{\cancel{x}\left(x+2-\frac{3}{x}\right)}{\cancel{x}\left(1-\frac{2}{x}\right)} = +\infty$$

($x+2-\frac{3}{x} \to \infty$, $1-\frac{2}{x} \to 0$)

oblique asymptote ? $y = ax + b$

$$a = \lim_{x\to+\infty} \frac{f(x)}{x} = \frac{x^2+2x-3}{(x-2)x} = \frac{x^2+2x-3}{x^2-2x}$$

$$= \frac{\infty}{\infty} \Rightarrow \frac{x^2\left(1+\frac{2}{x}-\frac{3}{x^2}\right)}{x^2\left(1-\frac{2}{x}\right)} = 1 \to a$$

($\frac{2}{x}, \frac{3}{x^2} \to 0$; $1-\frac{2}{x} \to 0$)

$$b = \lim_{x\to+\infty} f(x) - ax \Rightarrow \lim_{x\to+\infty} \frac{x^2+2x-3}{x-2} - 1x$$

$$= \frac{x^2+2x-3-x^2+2x}{x-2} = \frac{4x-3}{x-2} = \frac{\infty}{\infty} \Rightarrow$$

$$\Rightarrow \lim_{x\to+\infty} \frac{x\left(4-\frac{3}{x}\right)}{x\left(1-\frac{2}{x}\right)} = 4 \to b$$

($\frac{3}{x} \to 0$; $\frac{2}{x} \to 0$)

$y = x + 4$ oblique asymptote

- MAX and min

$$D\left(\frac{x^2+2x-3}{x-2}\right) = \frac{(2x+2)(x-2)-1(x^2+2x-3)}{(x-2)^2}$$

$(x-2)^2$ → always +

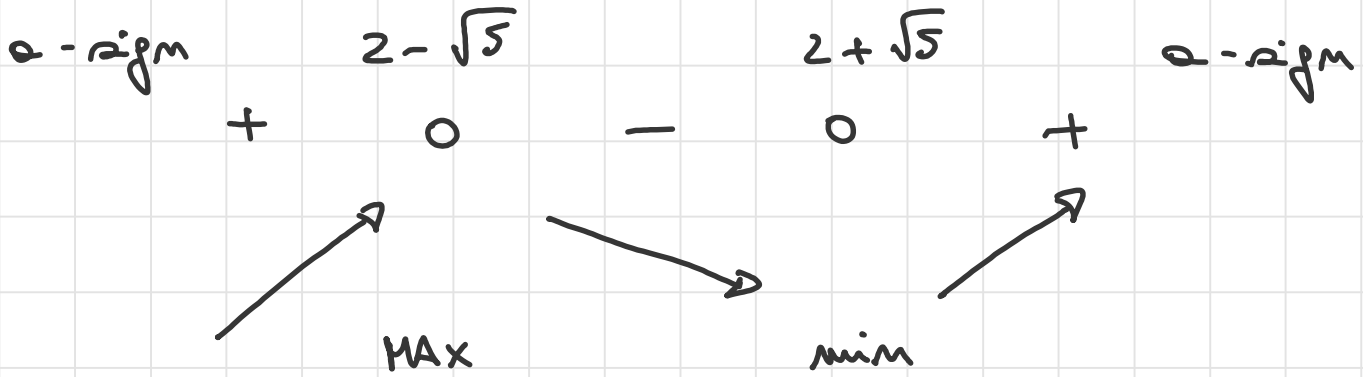
$$\Rightarrow 2x^2 - 4x + 2x - 4 - x^2 - 2x + 3 = x^2 - 4x - 1$$

$\Delta = 16 + 4 = 20 > 0 \Rightarrow$ 2 real solutions

$\Rightarrow x_{1/2} : \frac{4 \pm \sqrt{20}}{2} = \frac{4 \pm 2\sqrt{5}}{2} =$

20	2
10	2
5	5

$= 2 \pm \sqrt{5}$



7. Tom spends all his 72 weekly income on two goods, X and Y. His utility function is given by $U(X,Y) = 6X^{1/3}Y^{2/3}$. If $P_X = 8$ and $P_Y = 2$, how much of each good should he buy?

a. $X = 2; Y = 28$ **b.** $X = 3; Y = 24$ **c.** $X = 4; Y = 20$ **d.** $X = 5; Y = 16$

$$L = 6x^{1/3}y^{2/3} - \lambda(8x + 2y - 72)$$

$$\begin{cases} \frac{\partial L}{\partial x} = 0 \Rightarrow 6 \cdot \frac{1}{3} x^{-2/3} y^{2/3} - 8\lambda = 0 \\ \frac{\partial L}{\partial y} = 0 \Rightarrow 6 \frac{2}{3} x^{1/3} y^{-1/3} - 2\lambda = 0 \\ \frac{\partial L}{\partial \lambda} = 0 \Rightarrow 8x + 2y = 72 \quad (\text{v. di B.}) \end{cases}$$

Dalle prime due equazioni ricavo

$$\frac{2\left(\frac{y}{x}\right)^{2/3}}{2\left(\frac{x}{y}\right)^{1/3}} = \frac{8y}{2x}$$

Divido membro a membro

$$\frac{1}{2}\left(\frac{y}{x}\right)^{2/3}\cdot\left(\frac{y}{x}\right)^{1/3} = 4 \Rightarrow \quad y = 8x$$

$\Rightarrow$ sostituisco nel V. di B.

$$\Rightarrow \quad 8x + 2\cdot 8x = 72 \Rightarrow 24x = 72 \Rightarrow x = 3$$

$$y = 8\cdot 3 \Rightarrow \quad y = 24$$

8. Consider the following two-player game with normal form and find all Nash equilibria

		Player 2		
		C	D	E
Player 1	A	3 7	8 5	5 3
	B	6 8	4 6	3 1

Best reply ...

There is 1 Nash equilibrium : (B; C)

PLAYER 1

if P_2 plays C => P_1 prefers B(8) to A(7)
" " " D => " " B(6) to A(5)
" " " E => " " A(3) to B(1)

PLAYER 2

if P_1 plays A => P_2 prefers D(8) to C(3) and E(5)
" " " B => " " C(6) to D(4) and E(3)

Payoffs are both highlighted in (B, C)
the unique Nash equilibrium of the game

9. Consider the following financial operation and find the Net Present Value when the cost of capital is 5%

Year	Cash Flows
0	-3000
1	-1000
2	5000
4	2000

$$NPV = -3000 \quad \frac{-1000}{1+0.05} + \frac{5000}{(1+0.05)^2} + \frac{2000}{(1+0.05)^4}$$

$$= -3000 - 952,38 + 4535.15 + 1645.40$$

$$= 2228.17$$