

8. Consider the following two-player game with normal form and find all Nash equilibria

		Player 2		
		C	D	E
Player 1	A	2, 2	2, 1	3, 0
	B	1, 2	3, 3	2, 1

- a. (A, C); (B, D) b. (A, C) c. (B, D) d. (A, C); (A; D)

9. Consider the following financial operation and find the Net Present Value when the cost of capital is 10%

Year	Cash Flows
0	-3000
1	2000
3	1000
4	1000

- a. 340.12 b. 252.51 c. 1000.00 d. 403.22

$$y = x^3 - 8x^2 + 17x - 10$$

- Domain \Rightarrow polynomial $f. \Rightarrow x \in \mathbb{R}$
- Even or odd f ? $f(-x) = (-x)^3 - 8(-x)^2 + 17(-x) - 10$
 $= -x^3 - 8x^2 - 17x - 10$ NEITHER

• Intersections

if $x = 0 \Rightarrow y = -10$ $(0; -10)$

if $y = 0 \Rightarrow x^3 - 8x^2 + 17x - 10 = 0$

\Rightarrow Ruffini's rule \Rightarrow integer divisors of -10

$\Rightarrow \{ \pm 1; \pm 2; \pm 5; \pm 10 \}$

$x = 1 \Rightarrow 1 - 8 + 17 - 10 = 0$ \checkmark root

$(x - 1)(\dots)$

	1	-8	17	-10
1	↓			
	1	-7	10	10
	1	-7	10	//

$F_1 \quad F_2$
 $(x-1)(x^2 - 7x + 10)$



$\Delta = 49 - 40 = 9 > 0$

2 real solutions

$$x_{1/2} : \frac{7 \pm \sqrt{9}}{2} \begin{cases} \rightarrow \frac{7-3}{2} = 2 \\ \rightarrow \frac{7+3}{2} = 5 \end{cases}$$

$$(0; -10) ; (1; 0) ; (2; 0) ; (5; 0)$$

- y positive if ... roots already found

		1		2		5		
F_1	-	0	+	+	+	+	+	
F_2	<i>a-sign</i>	+	+	+	0	-	0	+
$f(x)$	-	0	+	0	-	0	+	
		*	*	*	*	*	*	

$$y > 0 \quad \text{if} \quad 1 < x < 2 \quad \cup \quad x > 5$$

- Asymptotes \Rightarrow NOTE (polynomial f .)

$$\lim_{x \rightarrow \pm\infty} x^3 - 8x^2 + 17x - 10 = \pm\infty$$

oblique asymptote? $y = ax + b$

$$a = \lim_{x \rightarrow \infty} \frac{f(x)}{x} = \frac{x^3 + \dots}{x} = \infty \quad \begin{matrix} \nearrow \\ \text{a deve} \\ \text{essere} \\ \text{finito!} \end{matrix}$$

• MAX and min

$$D(x^3 - 8x^2 + 17x - 10)$$

$$\Rightarrow 3x^2 - 16x + 17 = 0 \Rightarrow$$

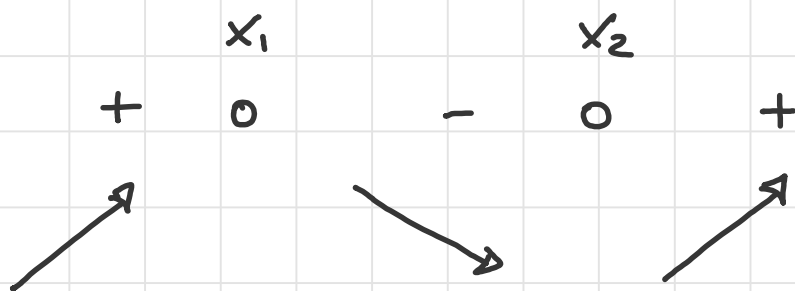


$$\Delta = 256 - 204 = 52 > 0 \Rightarrow 2 \text{ real sol.}$$

$$x_{1/2} : \frac{16 \pm \sqrt{52}}{6} \Rightarrow \begin{array}{r|l} 52 & 2 \\ 26 & 2 \\ 13 & 13 \end{array}$$

$$\Rightarrow \sqrt{52} = 2\sqrt{13}$$

$$x_{1/2} : \frac{16 \pm 2\sqrt{13}}{6} \begin{array}{l} \nearrow x_1 = \frac{8}{3} - \frac{\sqrt{13}}{3} \sim 1,46 \\ \searrow x_2 = \frac{8}{3} + \frac{\sqrt{13}}{3} \sim 3,86 \end{array}$$



$$\text{MAX at } \frac{8}{3} - \frac{\sqrt{13}}{3}$$

$$\text{min at } \frac{8}{3} + \frac{\sqrt{13}}{3}$$

7. Tom spends all his 120 weekly income on two goods, X and Y. His utility function is given by $U(X, Y) = 2X^2Y$. If $P_X = 10$ and $P_Y = 2$, how much of each good should he buy?

a. $X = 5; Y = 35$

b. $X = 6; Y = 30$

c. $X = 7; Y = 25$

d. $X = 8; Y = 20$

$$L = 2x^2y - \lambda (10x + 2y - 120)$$

$$\frac{\partial L}{\partial x} = 0 \Rightarrow 4xy - 10\lambda = 0$$

$$\frac{\partial L}{\partial y} = 0 \Rightarrow 2x^2 - 2\lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow 10x + 2y = 120 \quad (\text{v. di B.})$$



Dalle prime due ricavata

$$\begin{cases} \frac{4xy}{2x^2} = \frac{10\lambda}{2\lambda} \\ \frac{2y}{x} = 5 \end{cases}$$

Divido membro a membro...

$$2 \frac{y}{x} = 5 \Rightarrow y = \frac{5}{2}x$$

⇒ Sostituisco nel vincolo di Bilancio

$$\Rightarrow 10x + \cancel{2} \cdot \frac{5}{\cancel{2}}x = 120 \Rightarrow 15x = 120 \Rightarrow x = 8 ; y = \frac{5}{2} \cdot 8 = 20$$

$y = 20$

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		Player 2		
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	B	1	(3)	2

Best reply ...

=> There are 2 Nash equilibria : (A;C) ; (B;D)

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0	-3000
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$$NPV = -3000 + \frac{2000}{1+0,1} + \frac{1000}{(1+0,1)^3} + \frac{1000}{(1+0,1)^4}$$

$$= -3000 + 1818,18 + 751,31 + 683,01$$

$$= 252,51$$

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F_1	-	0	+	+	+	+	+	
F_2	$\xrightarrow{\text{e-sign}}$	+	+	+	0	-	0	+
$f(x)$	-	0	+	0	-	0	+	
		$\xrightarrow{*}$	$\xrightarrow{*}$		$\xrightarrow{*}$	$\xrightarrow{*}$		

$$y > 0 \quad \text{if} \quad 1 < x < 2 \quad \cup \quad x > 5$$

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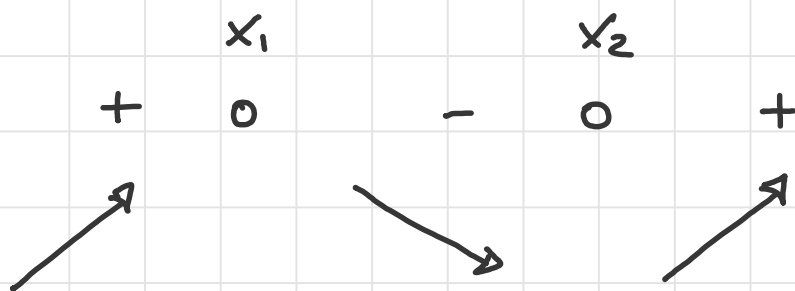


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Best reply ...

\Rightarrow There are 2 Nash equilibria : (A;C) ; (B;D)

PLAYER 1

if P_2 plays C $\Rightarrow P_1$ prefers A(2) to B(1)
 " " " D \Rightarrow " " B(3) to A(2)
 " " " E \Rightarrow " " A(3) to B(2)

PLAYER 2

if P_1 plays A $\Rightarrow P_2$ prefers C(2) to D(1) and E(0)
 " " " B \Rightarrow " " D(3) to C(2) and E(1)

there are 2 Nash equilibria : (A, C) and (B, D),
 where the payoffs are both highlighted

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