Last Name _______________ First Name _______________ Identification Number _______________

Degree in Economics, Business and Financial Markets
English Laboratory of Mathematics for Economics – Prof. E. Colantonio
Test of xx/xx/xxxx

Time available: 90 minutes

You must answer at least 5 (of 9) questions correctly to pass the Test (wrong answers do not penalize)

Given the function $y = x^3 - 8x^2 + 17x - 10$

1. State the domain

a. $x \geq 0$ **b.** $x \geq 1$ **c.** $x \leq 1$ **d.** $x \in R$

2. Is the function even, odd, both or neither?

a. even **b.** odd **c.** both **d.** neither

3. Find the points at which the function cuts the x and y axes

a. $(0;-10);(1;0);(2;0);(5;0)$ **b.** $(0;-10);(2;0);(5;0)$ **c.** $(0;-10);(1;0)\ (2;0);(6;0)$ **d.** $(0;-10);(2;0)\ (3;0);(5;0)$

4. y is positive if…

a. $1 < x < 2 \cup x > 6$ **b.** $1 < x < 3 \cup x > 6$ **c.** $1 < x < 2 \cup x > 5$ **d.** $0 < x < 2 \cup x > 5$

5. Find any asymptotes

a. $x = 0$ **b.** $y = -4$ **c.** $y = x - 2$ **d.** none

6. Find any minima (min) and maxima (MAX) for the function

a. $MAX\ at\ \left(\frac{8}{3} - \frac{\sqrt{13}}{3}\right)$ $min\ at\ \left(\frac{8}{3} + \frac{\sqrt{13}}{3}\right)$ **b.** $MAX\ at\ \left(8 - \frac{\sqrt{13}}{3}\right)$ $min\ at\ \left(8 + \frac{\sqrt{13}}{3}\right)$ **c.** $MAX\ at\ \left(\frac{1}{3} - \frac{\sqrt{13}}{3}\right)$ $min\ at\ \left(\frac{1}{3} + \frac{\sqrt{13}}{3}\right)$ **d.** $MAX\ at\ \left(\frac{8}{3} - \sqrt{13}\right)$ $min\ at\ \left(\frac{8}{3} + \sqrt{13}\right)$

7. Tom spends all his 120 weekly income on two goods, *X* and *Y*. His utility function is given by $U(X,Y) = 2X^2Y$. If $P_X = 10$ and $P_Y = 2$, how much of each good should he buy?

a. $X = 5; Y = 35$ **b.** $X = 6; Y = 30$ **c.** $X = 7; Y = 25$ **d.** $X = 8; Y = 20$

8. Consider the following two-player game with normal form and find all Nash equilibria

		Player 2		
		C	D	E
Player 1		2	1	0
	A	2	2	3
		2	3	1
	B	1	3	2

a. (A, C); (B, D) **b.** (A, C) **c.** (B, D) **d.** (A, C); (A; D)

9. Consider the following financial operation and find the Net Present Value when the cost of capital is 10%

Year	Cash Flows
0	-3000
1	2000
3	1000
4	1000

a. 340.12 **b.** 252.51 **c.** 1000.00 **d.** 403.22

$y = x^3 - 8x^2 + 17x - 10$

- Domain $\Rightarrow$ polynomial f. $\Rightarrow$ $x \in \mathbb{R}$
- Even or odd f ? $f(-x) = (-x)^3 - 8(-x)^2 + 17(-x) - 10$

$= -x^3 - 8x^2 - 17x - 10$ NEITHER

- Intersections

if $x = 0 \Rightarrow y = -10$ $(0; -10)$

if $y = 0 \Rightarrow x^3 - 8x^2 + 17x - 10 = 0$

$\Rightarrow$ Ruffini's rule $\Rightarrow$ integer divisors of -10

$\Rightarrow \{\pm 1; \pm 2; \pm 5; \pm 10\}$

$x = 1 \Rightarrow 1 - 8 + 17 - 10 = 0$ ✓ root

$(x-1)(\ldots)$

	1	−8	17	−10
1		1	−7	10
	1	−7	10	//

$\overset{F_1}{(x-1)}\overset{F_2}{(x^2 - 7x + 10)}$

$\Delta = 49 - 40 = 9 > 0$
2 real solutions

$$x_{1/2} : \frac{7 \pm \sqrt{9}}{2} \begin{cases} \frac{7-3}{2} = 2 \\ \frac{7+3}{2} = 5 \end{cases}$$

$(0; -10)$; $(1; 0)$; $(2; 0)$; $(5; 0)$

- y positive if ...   roots already found

		1		2		5	
F_1	$-$	0	$+$	$+$	$+$	$+$	$+$
F_2 (a-sign … a-sign)	$+$ $+$		$+$	0	$-$	0	$+$
$f(x)$	$-$	0	$+$	0	$-$	0	$+$

$y > 0$ if $1 < x < 2 \ \cup \ x > 5$

- Asymptotes $\Rightarrow$ NONE (polynomial f.)

$$\lim_{x \to \pm\infty} x^3 - 8x^2 + 17x - 10 = \pm\infty$$

oblique asymptote ? $y = ax + b$

$$a = \lim_{x \to \infty} \frac{f(x)}{x} = \frac{x^3 + \dots}{x} = \infty$$

a deve essere finito!

- MAX and min

$D\left(x^3 - 8x^2 + 17x - 10\right)$

$\Rightarrow \quad 3x^2 - 16x + 17 = 0 \quad \Rightarrow$

$\Delta = 256 - 204 = 52 > 0 \quad \Rightarrow \quad$ 2 real sol.

$x_{1/2} : \dfrac{16 \pm \sqrt{52}}{6} \quad \Rightarrow \quad \begin{array}{r|l} 52 & 2 \\ 26 & 2 \\ 13 & 13 \end{array}$

$\Rightarrow \quad \sqrt{52} = 2\sqrt{13}$

$x_{1/2} : \dfrac{16 \pm 2\sqrt{13}}{6}$

$x_1 = \dfrac{8}{3} - \dfrac{\sqrt{13}}{3} \sim 1{,}46$

$x_2 = \dfrac{8}{3} + \dfrac{\sqrt{13}}{3} \sim 3{,}86$

	x_1		x_2	
+	0	−	0	+
↗		↘		↗

MAX at $\dfrac{8}{3} - \dfrac{\sqrt{13}}{3}$ min at $\dfrac{8}{3} + \dfrac{\sqrt{13}}{3}$

7. Tom spends all his 120 weekly income on two goods, X and Y. His utility function is given by $U(X, Y) = 2X^2Y$. If $P_X = 10$ and $P_Y = 2$, how much of each good should he buy?

a. $X = 5; Y = 35$ **b.** $X = 6; Y = 30$ **c.** $X = 7; Y = 25$ **d.** $X = 8; Y = 20$

$$L = 2x^2Y - \lambda(10x + 2Y - 120)$$

$$\frac{\partial L}{\partial x} = 0 \Rightarrow 4xY - 10\lambda = 0$$

$$\frac{\partial L}{\partial Y} = 0 \Rightarrow 2x^2 - 2\lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow 10x + 2Y = 120 \quad (\text{V. di B.})$$

Dalle prime due si ricava

$$\begin{cases} \cancel{4}^{2} \cancel{x} Y = \cancel{10}^{5} \cancel{\lambda} \\ \hline \cancel{2} x^{\cancel{2}} = \cancel{2} \cancel{\lambda} \end{cases}$$

Divido membro a membro...

$$2\frac{Y}{X} = 5 \Rightarrow Y = \frac{5}{2}X$$

$\Rightarrow$ Sostituisco nel Vincolo di Bilancio

$$\Rightarrow 10X + \cancel{2} \cdot \frac{5}{\cancel{2}}X = 120 \Rightarrow 15X = 120 \Rightarrow X = 8 \; ; \; Y = \frac{5}{\cancel{2}} \cdot \cancel{8}^{4} = 20$$

$Y = 20$

8. Consider the following two-player game with normal form and find all Nash equilibria

		Player 2		
		C	D	E
Player 1	A	2 , 2	2 , 1	3 , 0
	B	1 , 2	3 , 3	2 , 1

Best reply ...

=> There are 2 Nash equilibria : (A;C) ; (B;D)

9. Consider the following financial operation and find the Net Present Value when the cost of capital is 10%

Year	Cash Flows
0	-3000
1	2000
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$$NPV = -3000 + \frac{2000}{1+0,1} + \frac{1000}{(1+0,1)^3} + \frac{1000}{(1+0,1)^4}$$

$$= -3000 + 1818,18 + 751,31 + 683,01$$

$$= 252,51$$

$y = x^3 - 8x^2 + 17x - 10$

- Domain $\Rightarrow$ polynomial f. $\Rightarrow$ $x \in \mathbb{R}$
- Even or odd f ? $f(-x) = (-x)^3 - 8(-x)^2 + 17(-x) - 10$

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$(x-1)(\ldots)$

	1	-8	17	-10
1	↓	1	-7	10
	1	-7	10	//

F_1 F_2

$(x-1)(x^2 - 7x + 10)$

$\Delta = 49 - 40 = 9 > 0$

2 real solutions

$$x_{1/2}: \quad \frac{7 \pm \sqrt{9}}{2} \quad \begin{cases} \frac{7-3}{2} = 2 \\ \frac{7+3}{2} = 5 \end{cases}$$

$(0; -10)$; $(1; 0)$; $(2; 0)$; $(5; 0)$

- y positive if ... roots already found

		1		2		5	
F_1	$-$	0	$+$	$+$	$+$	$+$	$+$
	a-sign						a-sign
F_2	$+$	$+$	$+$	0	$-$	0	$+$
$f(x)$	$-$	0	$+$	0	$-$	0	$+$

$y > 0$ if $1 < x < 2 \;\cup\; x > 5$

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	x_1		x_2	
$+$	0	$-$	0	$+$
$\nearrow$		$\searrow$		$\nearrow$

MAX at $\dfrac{8}{3} - \dfrac{\sqrt{13}}{3}$ $\qquad$ min at $\dfrac{8}{3} + \dfrac{\sqrt{13}}{3}$

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$Y = 20$

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	B	1 , 2	3 , 3	2 , 1

Best reply ...

⇒ There are 2 Nash equilibria : (A;C) ; (B;D)

<u>PLAYER 1</u>

if P_2 plays C ⇒ P_1 prefers A(2) to B(1)
" " " D ⇒ " " B(3) to A(2)
" " " E ⇒ " " A(3) to B(2)

<u>PLAYER 2</u>

if P_1 plays A ⇒ P_2 prefers C(2) to D(1) and E(0)
" " " B ⇒ " " D(3) to C(2) and E(1)

there are 2 Nash equilibria : (A, C) and (B, D), where the payoffs are both highlighted

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